PREDICTED QUESTION PAPER – February/March 2023

CLASS: 12 SUB: MATHEMATICS(041) Marks: 80 Time: 3 hours

General Instructions:

1. **This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.**
2. **Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.**
3. **Section B has 5 Very Short Answer (VSA)-type questions of 2 mark each.**
4. **Section C has 6 Short Answer (SA)-type questions of 3 mark each.**
5. **Section D has 4 Long Answer (LA)-type questions of 5 mark each.**
6. **Section E has 3 sources based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.**

SECTION A

(Multiple Choice Questions) Each question carries 1 mark

1. If Matrix A = $\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = \lambda A$, then the value of λ is

   (a) 6 (b) 8 (c) 18 (d) 0

2. The Matrix A = $\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric. Then the value of a + b is

   (a) $\frac{6}{7}$ (b) $\frac{5}{6}$ (c) $\frac{7}{8}$ (d) $\frac{8}{9}$

3. If A is a 3 × 3 invertible matrix , where $det(A^{-1}) = (det A)^k$, then the value of k is

   (a)1 (b) 2 (c) -2 (d) -1

4. If A is a singular matrix , $A(adj\ A)$ is

   (a) Null Matrix (b)Scalar Matrix

   (c)Identity Matrix (d) None of these

5. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then α is equal

(a) ±1 (b) ±3 (c) ±2 (d) ±4

6. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then the value of λ is

(a) -2 (b)8 (c) -1 (d) +2

7. The two vectors $\hat{\imath} + \hat{\jmath}$ and $3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ represent the sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC. The length of the median through A is

(a) $2\sqrt{3}$ (b) $-2\sqrt{3}$ (c) $2\sqrt{2}$ (d) $-2\sqrt{2}$

8. The area of the parallelogram having the diagonal , $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ is

(a)$5\sqrt{3}$ (b) $4\sqrt{3}$ (c)$-5\sqrt{3}$ (d) $-4\sqrt{3}$

9. The Cartesian equation of a line are $x = ay + b$, $z = cy + d$. Its direction ratios are

(a) $1, 1, c$ (b) $a, 1, c$ (c) a, a, c (d) $1, a, c$

10. A bag contains 25 tickets, numbered from 1 to 25.A ticket is drawn and then another ticket is drawn without replacement. The probability that both tickets will show even numbers is

(a) $\frac{11}{50}$ (b)$\frac{12}{51}$ (c) $\frac{11}{51}$ (d)$\frac{12}{50}$

11. The sum of order and degree of the given differential equation is

$$2\frac{d^2y}{dx^2} + 3\sqrt{1 - \left(\frac{dy}{dx}\right)^2} - y = 0$$

(a) 5 (b) 6 (c) 4 (d)2

12. The integrating factor of the given differential equation is

$x\frac{dy}{dx} + y - x + xy\, cotx = 0$ is

(a) x cosx (b) x sinx (c) x tanx (d) x cot x

13. The value of $\int_0^{\frac{\pi}{2}} \log(tanx)\, dx$ is

(a) 1 (b) 0 (c) -1 (d) π

14. If $f'(x) = x - \frac{1}{x^2}$ and f(1)= $\frac{1}{2}$, then f(x) is

(a) $\frac{x^2}{2} + \frac{1}{x} - 1$ (b) $\frac{x^2}{2} - \frac{1}{x} - 1$

(c) $\frac{x^2}{2} + \frac{1}{x} + 1$ (d) $-\frac{x^2}{2} + \frac{1}{x} - 1$

15. If f(x) = $\begin{cases} \frac{x-4}{|x-4|} + a & ; if\ x < 4 \\ a + b & ; if\ x = 4 \\ \frac{x-4}{|x-4|} + b & ; if\ x > 4 \end{cases}$ *is continuous at* $x = 4$

Then the value of a and b is

(a) a = 1 , b= 1 (b) a = 1 , b = - 1

(c) a = -1 , b = 1 (d) a = -1, b = -1

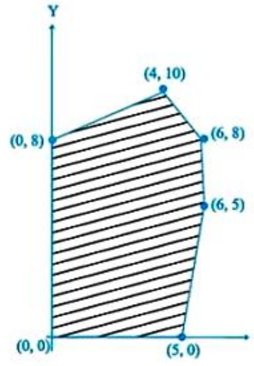
16. If y = sin (log x), then $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is equal to

(a) y (b) – y (c) x (d)-x

17. If the objective function for an LPP is Z = 3x + 4y and the corner points for unbounded feasible region are (9,0), (4,3) , (2,5) and (0,8),then the minimum value of Z occurs at

(a) (0,8) (b)(2,5) (c) (9,0) (d) (4,3)

18. In the given graph , the feasible region for a LPP is shaded. The objective function $Z = 2x - 3y$, will be minimum at



(a) (4,10) (b) (6,8) (c) (0,8) (d) (6,5)

ASSERTION – REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

19. Assertion (A): The domain of the function $\sin^{-1}(2x-1)$ is $[0,1]$.

Reason(R): The domain of the function $\sin^{-1} x$ is $[-1,1]$.

20. Assertion (A): The angle between two lines whose direction ratios are proportional to 1,1,2 and $(\sqrt{3}-1),(-\sqrt{3}-1),4$ is $\frac{\pi}{3}$.

Reason(R):Angle between the two given lines $\vec{r}=\vec{a}_1+\lambda\overrightarrow{m_1}$, $\vec{r}=\vec{a}_2+\mu\overrightarrow{m_2}$ is same as the angle between $\overrightarrow{m_1}$ and $\overrightarrow{m_2}$ where $\cos\theta=\frac{\overrightarrow{m_1}.\overrightarrow{m_2}}{|\overrightarrow{m_1}||\overrightarrow{m_2}|}$

SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 marks each)

21. Evaluate $tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+tan^{-1}(-\sqrt{3})+tan^{-1}\left(sin\left(-\frac{\pi}{2}\right)\right)$

OR

Let A = R – {3} and B = R – {1}, consider the function $f: A \to B$ defined by

$f(x)=\frac{x-2}{x-3}$. Is f one-one, onto or both?

22. If the area of circle increases at a uniform rate, then prove that the perimeter varies inversely as the radius.

23. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3).

OR

Let $\vec{a}=2\hat{\imath}+\hat{k}$, $\vec{b}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{c}=4\hat{\imath}-3\hat{\jmath}+7\hat{k}$ be three vectors. Find a vector $\vec{r}$ which satisfies $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$.

24. If x = $\tan\left(\frac{1}{a}\log y\right)$, show that $(1+x^2)\frac{d^2y}{dx^2}+(2x-a)\frac{dy}{dx}=0$.

25. If $\vec{a}=\hat{\imath}+2\hat{\jmath}+\hat{k}$; $\vec{b}=2\hat{\imath}+\hat{\jmath}$ and $\vec{c}=3\hat{\imath}-4\hat{\jmath}-5\hat{k}$, then find a unit vector perpendicular to both of the vectors $(\vec{a}-\vec{b})$ *and* $(\vec{c}-\vec{b})$.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Integrate the function $\frac{\cos(x+a)}{\sin(x+b)}$ with respect to x.

27. There is a group of 50 people who are patriotic out of which 20 believe in non-violence. Two persons are selected at random out of them. Write the probability distribution for the selected persons who are non-violent. Also, find its Mean/Expectation.

OR

A factory has three machines X , Y and Z producing 1000, 2000 and 3000 bolts per day respectively The machine X produces 1%, Y produces 1.5% and Z produces 2% defective items. The management collected some information for improvement in efficiencies.

(i) Find the conditional probability that defective item is produced given that it was produced by machine Y?

(ii) Find the probability that machine Z produced the product which was defective?

(iii)Find the total probability of defect in process of producing bolt?

28. Evaluate $\int_1^4(|x-1|+|x-2|+|x-4|)\,dx$

OR

Evaluate $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{x\sin x}{e^x+1}\,dx$

29. Solve the differential equation; $(1+y^2)dx = (\tan^{-1} y - x)dy,\ when\ y = 0, x = 0.$

OR

Find the particular solution of the differential equation $(x^2+xy)dy = (x^2+y^2)dx$ **given that** $= 0\ when\ x = 1$ **.**

30. Solve the following linear programming problem graphically.

Maximize and minimize $Z = 5x + 2y$

Subject to constraints; $x - 2y \le 2;\ 3x + 2y \le 12;\ -3x + 2y \le 3\ ;\ x \ge 0; y \ge 0$

31. Find $\int \frac{\sqrt{x}}{\sqrt{x}-\sqrt[3]{x}}$ **dx**

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Find the area of the region bounded by the parabola $y^2 = 2x$ **and straight line** $x - y = 4.$

33. Prove that the relation R on the set $N \times N$ **defined by** $(a,b)R(c,d) \Leftrightarrow a + d = b + c$ **for all** $(a,b),(c,d) \in N \times N$ **is an equivalence relation.**

OR

Let the relation S in the set of all real numbers R, be defined as $aSb \Leftrightarrow 1 + ab > 0$ **for all a, b**ϵR**. Is relation S an equivalence relation?**

34. Two cats are walking along the path $\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ **and** $\vec{r} = (5\hat{\imath} - 2\hat{\jmath}) + \mu(\widehat{3\imath} + 2\hat{\jmath} + 6\hat{k})$ **respectively. Show that the paths taken are intersecting. Hence, find the point of intersection.**

OR

Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k}$ **and** $\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}.$

35. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ **and use it to solve the system of equations** $x - y + z = 4\ ;\ x - 2y - 2z = 9\ ; 2x + y + 3z = 1.$

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. A plant nursery is an establishment that raises, propagates, multiples, sells seedling, saplings and other planting materials for planting.
It is imperative to give the young seedlings special attention in the first few weeks after germination. Young and tender seedlings growing in nursery beds requires enough sunlight, nutrition and more economical to care for in small area than in the main planting field.

The relation between the heights of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight.
Based on the above information, answer the following,

(i) Find the rate of growth of the plant with respect to number of days exposed to sunlight and also find its critical point?

(ii) What will be height of the plant after 2 days?

(iii) Find the points of local maximum/local minimum, if any , in the interval (0,5) as well as the points of absolute maximum /absolute minimum in the interval [0,5]. Also, find the corresponding local maximum/local minimum and the absolute maximum /absolute minimum values of the function.

OR

(iii) Find the interval in which the plant is strictly increasing/ strictly decreasing in its growth.

37. **Apple designs and manufacturers mobile communications and media devices, personal computers and portable music players. The company also sells a range of related software, services, accessories, networking solutions, music and video streaming as well as third party digital content and applications. Apple is best known for its series of iPhones, iPads and Mac personal computers.**

The total profit function of the company is given by $P(x) = -5x^2 + 125x + 37500$ **where x is the production of the company.**

Based on the above information,

(i) **What will be the production when the profit is maximum?**

(ii) **What will be maximum profit?**

(iii) **What will be the production of the company when the profit is ₹ 38250?**

OR

(iii) When the production is 2 units what will be the profit of the company?

38. **Shooting sports is a group of competitive and recreational sporting activities. Target shooting with rifle includes a gun, especially one fired from shoulder level, having a long spirally grooved barrel.**

A coach is training 3 players. He observes that the player can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots.

(i) **What will be probability that any two of A, B and C will hit the target?**

(ii) **What is the probability that at least one of A, B or C will hit the target?**

<u>All the best!</u>